
Query-limited Black-box Attacks to Classifiers

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Abstract

1 We study black-box attacks on machine learning classifiers where each query
2 to the model incurs some cost or risk of detection to the adversary. We focus
3 explicitly on minimizing the number of queries as a major objective. Specifically,
4 we consider the problem of attacking machine learning classifiers subject to a
5 budget of feature modification cost while minimizing the number of queries, where
6 each query returns only a class and confidence score. We describe an approach
7 that uses Bayesian optimization to minimize the number of queries, and find that
8 the number of queries can be reduced to approximately one tenth of the number
9 needed through a random strategy for scenarios where the feature modification cost
10 budget is low.

11 1 Introduction

12 Recent works reveal the vulnerabilities of current machine learning models to carefully-crafted
13 adversarial examples [1, 2, 3, 4]. In many scenarios, complete model information is not available to
14 the attacker and hence it is important to study black-box attacks, where the attackers do not have full
15 knowledge of the model but only some way of interacting with it as a black box. In this work, we
16 focus on attacks where only query access to the model is available and each query result consists of a
17 predicted class and confidence score.

18 Since queries to the model are costly, attackers are motivated to minimize the number of queries
19 needed when interacting with the model. For example, if the adversary is attempting to craft messages
20 that evade a spam email detection systems, each query to the underlying classification model involves
21 sending an email and possibly poisoning an account, so the adversary is not be able to afford large
22 number of email queries [5]. Hence, our problem is: find an adversarial example that satisfies the
23 constraints on feature modification with the minimal number of queries. This problem can be cast as
24 a constrained optimization problem:

$$\min Q(\mathbf{x}) \quad \text{s.t. } f(\mathbf{x}) \neq f(\mathbf{x}^A) \wedge c(\mathbf{x}, \mathbf{x}^A) \leq C \quad (1)$$

25 where $Q(\mathbf{x})$ denotes the total number of queries consumed in searching for an adversarial example for
26 seed sample \mathbf{x} , $f(\mathbf{x})$ denotes the prediction label of instance \mathbf{x} , $c(\mathbf{x}, \mathbf{x}^A)$ denotes feature modification
27 cost, where \mathbf{x}^A is the original instance, and C is the budget for feature modification. In this paper, we
28 use the L_1 -norm to measure cost, $c(\mathbf{x}, \mathbf{x}^A) = \|\mathbf{x} - \mathbf{x}^A\|_1$, as is commonly used in the text domain.

29 This optimization problem is highly intractable as we do not have a closed form expression for
30 function $Q(\mathbf{x})$. Further, $f(\mathbf{x})$ is unknown since we only have black-box access to the machine
31 learning model. Because of high intractability of the problem in Eq. (1), it is important for us to apply
32 some transformations to make the whole problem tractable. In particular, we convert the constrained
33 optimization form into one amenable to Bayesian optimization (Section 3).

34 The main contributions of our work are introducing a new formulation for black-box adversarial
35 machine learning where minimizing query numbers and proposing a query-minimizing black-box
36 attack strategy based on Bayesian optimization. Section 4 reports on preliminary experiments using
37 these techniques to generate spam messages that evade a black-box detector.

38 **Related Work** Prior works have studied black-box attacks on machine learning classifiers in two
39 categories: *substitute model attacks* and numerical *approximation method-based attacks*. The first
40 type of attack uses query responses obtained from the target model to train a substitute model, and
41 then generates adversarial examples for that substitute model. Adversarial examples produced this
42 way are transferable and often effective against the original model [6, 7, 8]. The drawback of the
43 substitute model is it will suffer from the transfer loss as not all adversarial examples can transfer
44 from one model to another model [9]. Also, the number of training instances needed to produce
45 an effective substitute model may be very large. One recent work in [10] adopts slightly different
46 strategy by learning a separate attacker model, which is trained to produce adversarial samples to the
47 target model. However, the query number minimization is still not explicitly considered in [10].

48 Another line of work, introduced by Chen et al. [9], is to apply some numerical approximation to
49 model gradient calculation to support known white-box attack strategies. This approximates gradient
50 information by the symmetric difference quotient and further utilizes the Carlini and Wagner attack
51 [11] to generate adversarial examples. This approach requires a large number of queries since the
52 gradient needs to be calculated in each step and each gradient estimation requires many model
53 evaluations because of the high-dimensional feature space.

54 Previous papers on black-box attacks do not explicitly consider minimizing the total number of model
55 interactions. A closely related work by Li and Vorobeychik [5] considers the spam email setting and
56 sets a bound on the total number of queries and feature modification cost. The attacker then applies a
57 query strategy to find adversarial examples. However, this work only considers linear classifiers. In
58 contrast, our work applies to arbitrary (continuous) classifiers, including neural networks and other
59 linear models.

60 2 Background on Bayesian Optimization

Bayesian optimization (BO) is a derivative free strategy for global optimization of black-box functions [12, 13, 14]. The Bayesian optimization problem can be formulated as:

$$\min g(\mathbf{x}) \quad \text{s.t. } h(\mathbf{x}) \leq 0.$$

61 where $g(\mathbf{x})$ is an unknown function and $h(\mathbf{x})$ can either be known or unknown. Unlike traditional
62 optimization algorithms, BO does not depend on gradient or Hessian information; instead, it works
63 by querying the function value of a point in each step of the interactive optimization process [12]. As
64 queries to $g(\mathbf{x})$ are assumed to be costly, the algorithm minimizes the total number of queries used in
65 the whole search process.

66 Since the objective function is unknown, a *prior* over the function is assumed to be known, e.g.,
67 Gaussian prior [15] is commonly used [12, 14, 13]. With the defined priors and current observations,
68 the *posterior probability* of next function value can be defined. With the posterior probability
69 distribution, an *acquisition function*, $\text{Acq}(\mathbf{x})$ is then defined to capture an *exploration-exploitation*
70 trade-off in determining the next query point [16, 17]. Points with larger $\text{Acq}(\mathbf{x})$ values are likely to
71 have smaller $g(\mathbf{x})$ values. Thus, we prefer to query points where $\text{Acq}(\mathbf{x})$ is large. Since the goal for
72 each step is to select a point that maximizes the current acquisition function, the whole optimization
73 process heuristically minimizes number of interactions needed to find a solution. Convergence rate of
74 Bayesian optimization can be referred to [16, 17].

75 Exploration prefers points where the uncertainty is high, while exploitation prefers points where
76 the objective function value is low (for minimization problems). After each function evaluation, the
77 acquisition function is updated along with the posterior probabilities. For this work, we use the upper
78 confidence bound (UCB) selection criterion in selecting the specific acquisition function type. As
79 we assume the unknown function value $g(\mathbf{x})$ at point \mathbf{x} follows Gaussian distribution, we obtain the
80 closed form expression of the acquisition function (UCB) for point \mathbf{x} as $\text{Acq}(\mathbf{x}) = \mu(\mathbf{x}) + \kappa\sigma(\mathbf{x})$,
81 where $\sigma(\mathbf{x})$ and $\mu(\mathbf{x})$ are variance and mean functions, respectively, at point \mathbf{x} and κ is a constant.
82 Brouchu et al.'s tutorial [12] provides more details regarding different types of acquisition functions
83 and closed form expressions for $\mu(\mathbf{x})$, $\sigma(\mathbf{x})$. Once the query result $g(\mathbf{x}_t)$ of the point \mathbf{x}_t is returned,
84 the BO framework updates its belief about the unknown function distribution and the whole procedure
85 iterates until termination condition is satisfied.

86 3 Minimizing Queries with Bayesian Optimization

87 As discussed in Section 1, we face two major challenges: no closed form expression for function
 88 $Q(\mathbf{x})$ and an unknown constraint in $f(\mathbf{x})$, where only queries to $f(\mathbf{x})$ are allowed. We handle the
 89 unknown constraint by following the approach used by Carlini and Wagner [11, 1] by moving the
 90 intractable classification label constraint into the objective function. Since we do not know $f(\mathbf{x})$,
 91 we transform the constraint of $f(\mathbf{x}) \neq f(\mathbf{x}^A)$ as minimizing the probability of \mathbf{x} having same label
 92 with \mathbf{x}^A . In order to minimize the total number of queries, as outlined in the objective function of
 93 Eq. (1), we adopt a heuristic strategy for minimization. Namely, in each query step, we use our
 94 query history to select the apparently best point for solving the optimization problem. Hence, specific
 95 to our problem, in each query step, we find the best point for minimizing $\Pr[f(\mathbf{x}) == f(\mathbf{x}^A)]$ and
 96 consequently, the whole optimization process eventually minimizes function $Q(\mathbf{x})$ (i.e. the total
 97 number of queries). Our query step will terminate once we have found a valid instance whose label is
 98 different from \mathbf{x}^A . The problem can be mathematically formulated as:

$$\min \Pr[f(\mathbf{x}) == f(\mathbf{x}^A)] \text{ s.t. } c(\mathbf{x}, \mathbf{x}^A) \leq C \quad (2)$$

99 To solve the problem in Eq. (2), we adopt the Bayesian optimization framework which is well-
 100 suited for solving an unknown function (in our case, $\Pr[f(\mathbf{x}) == f(\mathbf{x}^A)]$) minimization with a
 101 minimal number of queries (i.e., minimizing $Q(\mathbf{x})$). Note that $c(\mathbf{x}, \mathbf{x}^A)$ is a function known to the
 102 adversary (i.e., L_1 -norm constraint). We now have a Bayesian optimization problem with an unknown
 103 objective and known constraint. We use Upper Confidence Bound (UCB) as the acquisition function
 104 ($\text{Acq}(\mathbf{x})$) and in each step we select the point that maximizes $\text{Acq}(\mathbf{x})$ with respect to the constraint
 105 $c(\mathbf{x}, \mathbf{x}^A) \leq C$.

106 We apply the DIRECT algorithm [18] to solve acquisition function maximization problem in Eq. (2).
 107 DIRECT is a well-known algorithm for solving global optimization problems. To increase the robust-
 108 ness of the code when facing a small cost budget C , we applied DIRECT with minor modifications.
 109 DIRECT works by dividing a unit hypercube sequentially and evaluating function values in each of
 110 the sub-hyper-rectangles [18] and the initial point is center of the unit hypercube. Originally, each
 111 dimension value of this point was determined by the lower and upper bounds in that dimension.
 112 When C is very small and the initial center is too far away from initial point \mathbf{x}^A , it is very hard to
 113 find an instance within the feature cost budget (which will result in very long search time). Instead,
 114 we now take the initial point \mathbf{x}^A as the center of the unit hypercube such that we can always find
 115 instances that satisfy the feature modification cost constraint. Algorithm 1 summaries our Bayesian
 116 optimization algorithm; details regarding Gaussian process update can be found in Rasmussen [15].

Algorithm 1 Bayesian Optimization Based Black-box Attack

Input: $\mathbf{x}^A, C, f(\mathbf{x}^A), N$ (maximum number of iterations)

Output: \mathbf{x}^*

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1:  $\mathbf{x}^* = \mathbf{x}^A$ 
2: for  $t = 1, 2, \dots, N$  do
3:   Find  $\mathbf{x}_t$  by solving problem  $\mathbf{x}_t = \text{argmax } \text{Acq}(\mathbf{x} | D_{1:t-1}), \text{ s.t. } c(\mathbf{x}, \mathbf{x}^A) \leq C$ 
4:   Sample the objective function value:  $y_t = \Pr(f(\mathbf{x}_t) == f(\mathbf{x}^A))$ 
5:   if  $f(\mathbf{x}_t) \neq f(\mathbf{x}^A)$  then
6:     return  $\mathbf{x}^* = \mathbf{x}_t$ ;
7:   end if
8:   Augment the data  $D_{1:t} = \{D_{1:t-1}, (\mathbf{x}_t, y_t)\}$  and update the Gaussian Process and  $\text{Acq}(\mathbf{x})$ .
9: end for
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117 4 Evaluation

118 We have conducted preliminary experiments to evaluate the effectiveness of BO based black-box
 119 attacks using a spam email dataset. The attacker’s objective is to create a spam email, \mathbf{x}^* , that is
 120 misclassified by the unknown classifier but is within C of the original spam email \mathbf{x}^A .

121 **Spam Email Dataset** The dataset [19] contains 4601 records and each record holds 57 attributes.
 122 Among the 57 features, 2 of them are integers (we discard these two attributes as we currently only
 123 handle continuous features). Every email is labeled as either spam or normal. We randomly choose

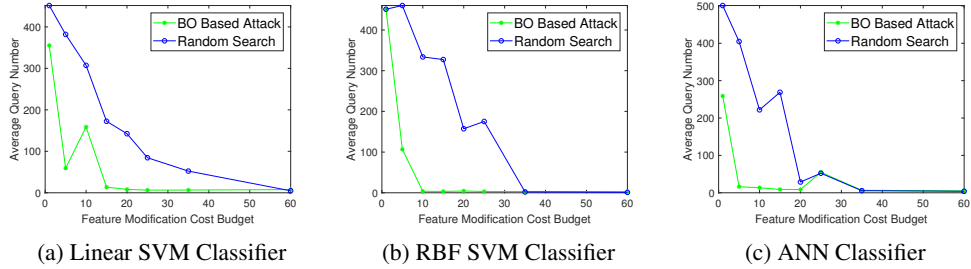


Figure 1: Average Query Number w.r.t Different Cost Budgets for Different Classifiers

124 3500 of the instances to train three different classifiers, and report the error rate on the remaining
 125 dataset. The original instance \mathbf{x}^A is randomly selected from the spam emails testing set.

126 **Classifier Models** We train both linear and RBF kernel probabilistic SVM, which achieve classifi-
 127 cation accuracy of 91% and 94% respectively. Details of transforming normal SVM into probabilistic
 128 SVM can be found in Platt [20]. We also train an ANN model with classification accuracy of 94%.

129 **Results and Discussion** We compare our results with random search, which randomly generates
 130 values for each dimension and terminates the search process when the class label is changed. Specifi-
 131 cally, we take the cost budget C and generate random samples whose L_1 -distance to \mathbf{x}^A is in the
 132 range of $(C - \epsilon, C)$. We set $\epsilon = 0.05$. Our assumption here is, having larger distance to the original
 133 instance can maximize the chance of flipping into opponent class, assuming the boundary of the
 134 classifier does not have a highly irregular shape.

135 For different classifiers, we compare the number of queries needed to find the first successful
 136 adversarial example for both algorithms (BO attack and random search) as we vary C from 1 to
 137 60. Note that, when C is extremely small, the chance of finding an adversarial example within the
 138 boundary is rare. Hence, we set some threshold values for both algorithms and once the iteration
 139 number exceeds the threshold, we take it as an indicator of non-existence of adversarial example. For
 140 our experiments, we set 50 as the upper limit for BO attack, and 500 for random search.

141 Figure 1 shows the average query number with respect to different feature modification cost budget
 142 C for the three models. We see significant reductions in the number of queries using the BO attack
 143 for all of the classifiers. Note that, the average number of queries shown here is a conservative
 144 estimate for the BO method, since we take all iterations of BO exceeding 50 as failure and compute
 145 the iteration number as 500 for fair comparison with random search method. For cases where C
 146 is small, our BO attack is substantially more efficient. For example, for the ANN Classifier with
 147 $C = 10$, it takes 16 queries to find the first successful adversarial example, compared to over 400 for
 148 the random search.

149 5 Conclusion

150 Our proposed black-box attack strategy considers the problem of generating adversarial example with
 151 minimum number of queries. We believe there are many scenarios where attackers will be limited in
 152 the number of interactions they have with a target model, so understanding the number of queries
 153 needed to find adversarial examples with high probability is an important problem. Our proposed
 154 Bayesian optimization approach shows promise in our preliminary experiments, and it is a general
 155 strategy that can be used against any classifier.

156 Our ongoing work will improve the BO attacks, compare with other black-box attacking methods, and
 157 test on data from different domains (e.g., image and text). We also note that, our approach can work
 158 for both targeted and untargeted attack. For targeted attacks, we simply set the objective function as
 159 maximizing $\Pr[f(\mathbf{x}) == y^*]$, where y^* is the target class.

160 For future directions, it is interesting to study the problem in a more restricted black-box case, where
 161 attackers only get classification labels. Another interesting direction is to provide a provable minimum
 162 modification to the original instance for adversarial sample generation in black-box setting.

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